TABLE V. Linear and volume compressibilities. Units: 10^{-13}cm²/dyne.

	Sb		Bi	
	This work	Bridgman[a] (Ref. 2)	ELR, recalculated	Bridgman[a] (Ref. 2)
k_t	4.1	5.40	6.38	3.59
k_z	17.5	16.84	18.07	16.13
k_v	25.8	27.64	30.83	29.31

[a] Isothermal values; isothermal-adiabatic correction is negligible.

bilities (Table V) within appropriately calculated tolerances.

Agreement with Kor's nominal value for c_{13} is not expected for it is extremely sensitive to the velocities. Kor calculates c_{13} from particular ELR velocities without first adjusting them to be compatible with the others. Consequently, our value for c_{13} is to be preferred.

c_{13}'s extreme sensitivity can be appreciated from the following formula:

$$c_{13}=\frac{\rho^2(v_{12a}{}^4+v_{14a}{}^4)-\rho^2(v_{9a}{}^4+v_{11a}{}^4)}{2c_{14}}-(c_{11}+2c_{44}),$$

where the symbols have their previously defined meanings. In principle, this expression can be used to calculate c_{13} directly, the extraneous root introduced by the quadratic already having been eliminated. We emphasize that properly calculated velocities and constants must be inserted unless one is willing to accept an uncertainty of 100% or more, and note that the value of c_{13} is, as it should be, independent of the convention used to determine the sign of c_{14} as the signs of the velocity function in the numerator and c_{14} change together. With this formula, it is necessary neither to employ the sign considerations outlined by Mayer and Parker[15] nor the conceivably less-discriminating strain-energy stability criteria.[16] An analogous expression for c_{13} in hexagonal systems, where c_{14} is identically zero, is not possible.

C. Elastic-Wave Refraction

In our attempt to understand v_{11}'s incompatibility for antimony, the 45° and 135° data were further analyzed in terms of the theory of plane elastic waves in aelotropic media.[17] Particle displacement and energy-flux directions, and the pure-mode direction in the Y-Z mirror plane, are calculated for both antimony and bismuth and compared with each other, and with the propagation and transducer-polarization directions. An outline of the calculation and the energy-flux expressions obtained are given in the Appendix; the results of this calculation, summarized in Fig. 2, are next discussed.

Our analysis shows that the unit displacement eigenvectors associated with the v_9 and v_{11} modes, A^9 and A^{11}, deviate by $-4°$ for antimony and $-5.2°$ for bismuth from the transducer polarization directions used to excite these modes. This small value is favorable for exciting the v_{11} mode in antimony, giving rise to the three well-defined pulses displayed by the oscilloscope. This same display obtains with either the 3-mm-diameter or the $\frac{1}{2}$-in.-square transducers. The corresponding deviations for A^{12} and A^{14} are $+14.6°$ for antimony and $+12.4°$ for bismuth. These deviations are not of a nature which would explain our egregious v_{11}, nor do the pure shear-mode directions in the Y-Z plane which are 117° for antimony and 107° for bismuth. However, the deviation of the energy flux, or ray velocity, from the normal is about 45°. For our dimensions, energy is deflected from a side before reaching the opposite reflecting face. Upon deflection the energy is refracted into spurious modes, giving rise to the pulses displayed. No pulse was found that corresponds with v_{11} calculated. This is due to the fact that in relation to the large intrinsic attenuation of antimony not enough energy flows along the wave-normal direction to reach the opposite face and to be echoed back to the transducer for detection.

We note that the smaller the flux deviation angle, the more numerous and better defined are the echoes, and that spurious pulses exist for almost every mode. Conical refraction effects along the triad axis, predicted by Waterman,[17] verified by Papadakis on rock salt and calcite,[18] and noticed by ELR in bismuth, did not interfere with obtaining decipherable echo patterns. For antimony the conical semi-angle in 28°40′; for bismuth, 32°30′.

Refraction effects were observed for either the 3-mm-diam or $\frac{1}{2}$-in-square transducers as sender-receiver. Many of the displays obtained with the larger one contained more spurious pulses than most displays obtained with the smaller one, depending apparently upon the propagation direction and the mode.

It is not possible for us to comment on the effects of the large deviation angles in bismuth since we do not have precise information on ELR's specimen geometry. We can only remark that the combined effects of specimen size, refraction, and attenuation are not as severe as they are in antimony. All of ELR's velocities are compatible and Eckstein[19] reports that his antimony echo displays are not as clean as they are for bismuth.

D. General Comments

Our experiment and analysis are based on plane elastic waves in extended media, and our specimens are

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